Automorphic Motives, Euler Systems & p-adic L-functions
3rd Nisyros Conference on Automorphic Representations & Related Topics
Nisyros island, Greece, 10–15 July 2017
Held at the Nisyros High School, Mandraki, Nisyros 85303, Greece

Schedule

Monday 10 July

8:45 Meet at Porfyris hotel to walk to the school.

9:15 Introductory remarks.

9:30 Eric Urban: "Euler systems via congruences between modular forms."

10:30 Coffee break

11:00 Valia Gazaki: "A Tate duality theorem for local Galois symbols."

12:00 Lunch.

14:00 Ming-Lun Hsieh: "p-adic triple product L-functions and applications."

15:15 Discussion panel: "New directions on the elliptic Stark conjecture", by Alan Lauder, Francesca Gatti and Marc Masdeu.

Tuesday 11 July

9:30 David Loeffler & Sarah Zerbes: "Euler systems for symplectic and unitary groups."

10:30 Coffee break

11:00 Matteo Longo: "The anticyclotomic Iwasawa main conjectures for elliptic curves."

12:00 Lunch.

14:00 Poster session: Kâzım Büyükboduk, Pierre Charollois, Christophe Cornut, Daniel Disegni, Preston Wake.

15:00 Discussion panel: "p-adic variation of modular forms, sheaves and connections", by Adrian Iovita, Daniel Barrera and Santiago Molina.

20:00 Conference dinner at Páloi.

Wednesday 12 July

9:30 Open discussion on p-adic L-functions and the geometry of the eigencurve.

12:00 Lunch at Emporiós.

16:00 Open discussion on the Euler systems of diagonal cycles and Beilinson–Flach elements.

Thursday 13 July

9:30 Yichao Tian: "The triple product Selmer group of an elliptic curve over a cubic real field."

10:30 Coffee break

11:00 Emmanuel Lecoutourier & Loïc Merel: "Higher Eisenstein elements in weight 2 and prime level."

12:00 Lunch.

14:00 Dimitar Jetchev: "Arithmetic Properties of Special Cycles on Unitary Shimura Varieties."

15:15 Discussion panel: "Venkatesh conjecture and triple-product L-functions", by Michael Harris, Henri Darmon and Victor Rotger.

Friday 14 July

9:30 Henri Darmon: "Generalised Kato classes and Eisenstein series: toward a p-adic analogue of a calculation of Harris and Venkatesh."

10:30 Coffee break

11:00 Chao Li: "Arithmetic fundamental lemma in the minuscule case."

12:00 Lunch.

14:00 Discussion panel: "The geometry of the eigencurve at weight one points", by Joël Bellaïche, Adel Betina, Mladen Dimitrov and Francesc Fité.

20:00 Dinner at Emboriós.

Saturday 15 July

10:00 Open discussion on Venkatesh's motivic conjecture.

Participants

- Daniel Barrera, UP Catalunya, Spain
- Joël Bellaïche, Brandeis, USA
- Adel Betina, UP Catalunya, Spain
- Kâzım Büyükboduk, Koç, Turkey
- Pierre Charollois, Paris 6/IMJ-PRG, France
- Masataka Chida, Tohoku University, Japan
- Christophe Cornut, CNRS/IMJ-PRG, France
- Henri Darmon, McGill, Canada
- Mladen Dimitrov, Lille, France
- Daniel Disegni, Paris-Sud, France
- Francesc Fité, UP Catalunya, Spain
- Francesca Gatti, UP Catalunya, Spain
- Valia Gazaki, Michigan, USA
- Valentin Hernandez, Paris 6, France
- Ming-Lun Hsieh, Academia Sinica, Taiwan
- Adrian Iovita, Concordia, Canada & Padova, Italy
- Dimitar Jetchev, EPFL, Switzerland
- Alan Lauder, Oxford, UK
- Arthur-César Le Bras, ENS, France

- Emmanuel Lecouturier, CNRS/IMJ-PRG, France
- Francesco Lemma, Paris 7/IMJ-PRG, France
- Chao Li, Columbia, USA
- David Loeffler, Warwick, UK
- Matteo Longo, Padova, Italy
- Marc Masdeu, UA Barcelona, Spain
- Loïc Merel, Paris 7/IMG-PRG, France
- Santiago Molina, CRM, Spain
- Óscar Rivero, UP Catalunya, Spain
- Joaquin Rodrigues, UCL, UK
- Giovanni Rosso, Cambridge, UK
- Yichao Tian, Bonn, Germany
- Eric Urban, Columbia, USA
- Preston Wake, UCLA, USA
- Carl Wang Erickson, Imperial College London, UK
- Sarah Zerbes, UCL, UK

Organizers

- Michael Harris, Paris 7/IMJ-PRG, France & Columbia University, USA
- Victor Rotger, UP Catalunya, Spain
- Yiannis Sakellaridis, Rutgers University–Newark, USA and National Technical University of Athens, Greece

[*]

[*]Lunch and dinner will be covered by ERC's Advanced Grant ERC-BSD GA 290766 held by Michael Harris at IHES (63,79%) and the Consolidator Grant ERC-BSD GA 682152 held by Victor Rotger at UPC (36,21%).

Abstracts

Henri Darmon

Title: Generalised Kato classes and Eisenstein series: toward a p-adic analogue of a calculation of Harris and Venkatesh.

Abstract: I will give an overview of conjectures formulated with Alan Lauder and Victor Rotger concerning the "generalised Kato classes" attached to a triple (f, g, h) of modular forms of weights $(2, 1, 1)$ and their p-adic dual exponentials and logarithms, with emphasis on the case where f is an Eisenstein series of weight two. The ultimate goal is to make a bridge with the tantalising calculations of Harris and Venkatesh experimentally verifying a conjecture of Venkatesh in the setting of GL(2).

Valia Gazaki

Title: A Tate duality theorem for local Galois symbols

Abstract: Let K be a p-adic field and M be a finite continuous $\mathrm{Gal}(\overline{\mathrm{K}}/\mathrm{K})$-module. Local Tate duality is a perfect duality between the Galois cohomology of M and the Galois cohomology of its dual module. In the special case when M is the module of the m-torsion points of an abelian variety A over K, Tate has a finer result. In this case the group $H^1(K, M)$ has a "significant subgroup", namely there is a map $A(K)/m \to H^1(K, M)$ induced by the Kummer sequence on A. Tate showed that under the perfect pairing for H^1, the orthogonal complement of $A(K)/m$ is the corresponding part, $A^\star(K)/m$, that comes from the points of the dual abelian variety $A^\star$ of A.

In this talk I will present an analogue of this classical result for H^2. We will show that the "significant subgroup" in this case is given by the image of a Galois symbol, $K(K; A, B)/m \to H^2(K, A[m] \otimes B[m])$, that generalizes the classical Galois symbol of the motivic Bloch–Kato conjecture, while the orthogonal complement under the Tate duality pairing will be given by an object of integral p-adic Hodge theory. We will then discuss applications to zero cycles on abelian varieties.

Ming-Lun Hsieh

Title: p-adic triple product L-functions and applications.

Abstract: I will report on the recent progress on p-adic (twisted) triple product L-functions attached to Hida families and discuss some applications.

Dimitar Jetchev

Title: Arithmetic Properties of Special Cycles on Unitary Shimura Varieties.

Abstract: We discuss recent progress on an ongoing project for constructing an Euler system from special cycles on certain unitary Shimura varieties. We explain the arithmetic properties of the special cycles and explain how these properties lead to norm-compatibility relations similar to the relations satisfied by classical cyclotomic units and Heegner points. Our results will cover both split and inert primes. These constructions are useful in the study of Selmer groups of Galois representations attached to certain automorphic forms on the corresponding unitary groups and higher-dimensional analogues of the Birch and Swinnerton-Dyer conjecture.

Emmanuel Lecoutourier & Loïc Merel

Title: Higher Eisenstein elements in weight 2 and prime level.

Abstract: In his classical work, Mazur considers the Eisenstein ideal I of the Hecke algebra $\mathbb{T}$ acting on cusp forms of weight 2 and level $\Gamma_0(N)$ where N is prime. When p is an Eisenstein prime, *i.e.* p divides the numerator of $\frac{N-1}{12}$, denote by $\mathbf{T}$ the completion of $\mathbb{T}$ at the maximal ideal generated by I and p. This is a $\mathbf{Z}_p$-algebra of finite rank $g_p \geq 1$ as a $\mathbf{Z}_p$-module.

Mazur asked what can be said about g_p. Merel was the first to study g_p. Assume for simplicity that $p \geq 5$. Let $\log : (\mathbf{Z}/N\mathbf{Z})^\times \to \mathbf{F}_p$ be a surjective morphism. Then Merel proved that

$$g_p \geq 2$$

if and only if

$$\sum_{k=1}^{\frac{N-1}{2}} k \cdot \log(k) \equiv 0 \text{ (modulo } p).$$

We prove that we have $g_p \geq 3$ if and only if

$$\sum_{k=1}^{\frac{N-1}{2}} k \cdot \log(k) \equiv \sum_{k=1}^{\frac{N-1}{2}} k \cdot \log(k)^2 \equiv 0 \text{ (modulo } p).$$

We also give a more complicated criterion to know when $g_p \geq 4$. Moreover, we prove *higher Eichler formulas*. More precisely, let

$$H(X) = \sum_{k=0}^{\frac{N-1}{2}} \binom{\frac{N-1}{2}}{k}^2 \cdot X^k \in \mathbf{F}_N[X]$$

be the classical Hasse polynomial. It is well-known that the roots of H are simple and in $\mathbf{F}_{N^2}^{\times}$. Let L be this set of roots. We prove that

$$\sum_{\lambda \in L} \log(H'(\lambda)) \equiv 4 \cdot \sum_{k=1}^{\frac{N-1}{2}} k \cdot \log(k) \text{ (modulo } p)$$

and, if $g_p \geq 2$,

$$\sum_{\lambda \in L} \log(H'(\lambda))^2 \equiv 4 \cdot \sum_{k=1}^{\frac{N-1}{2}} k \cdot \log(k)^2 \text{ (modulo } p) \ .$$

Chao Li
Title: Arithmetic fundamental lemma in the minuscule case.
Abstract: The arithmetic Gan-Gross-Prasad conjecture generalizes the Gross-Zagier formula to Shimura varieties associated to unitary or orthogonal groups. The arithmetic fundamental lemma (AFL), formulated by Wei Zhang in the unitary case, is a key local ingredient in the relative trace formula approach towards arithmetic GGP. The AFL compares arithmetic intersection numbers on Rapoport–Zink spaces with derivatives of orbital integrals. We prove an explicit formula for the arithmetic intersection numbers in both unitary and orthogonal cases, under a minuscule assumption. In particular, our work gives a new proof of the theorem of Rapoport–Terstiege–Zhang on the AFL. This is joint work with Yihang Zhu.

David Loeffler & Sarah Zerbes
Title: Euler systems for symplectic and unitary groups.
Abstract: We will describe the construction of two new Euler systems, living in the cohomology of Shimura varieties for the groups GSp(4) and GU(2, 1). This is joint work with Chris Skinner. The techniques used to prove the norm-compatibility relations in these Euler systems are closely related to restriction problems in representation theory and the local Gan—Gross–Prasad conjectures.

Matteo Longo
Title: The anticyclotomic Iwasawa main conjectures for elliptic curves.
Abstract: Let E be a rational elliptic curve, p a prime number, and K an imaginary quadratic field, of discriminant prime to the conductor of E, in which p is not ramified. Let K_∞ be the anticyclotomic $\mathbb{Z}_p$-extension of K. The anticyclotomic main conjecture relates, under suitable arithmetic conditions, a p-adic L-function (which interpolates central critical values of the L-function of E twisted by anticylclotomic characters) and the structure of the Selmer group of E over K_∞, viewed as a module over the Iwasawa algebra of K_∞/K. The formulation of the main conjecture depends on some arithmetic data: the behavior of p in K (split or inert) and the reduction type of E at p (good ordinary or supersingular). In a joint work, in progress, with M. Bertolini and R. Venerucci, for each of these situations we prove the relative version of the anticyclotomic Iwasawa main conjecture. I will try to explain some of the ideas underlying the proofs of these results.

Yichao Tian
Title: The triple product Selmer group of an elliptic curve over a cubic real field.
Abstract: Let F be a cubic totally real field, and E/F be a modular elliptic curve. We consider its triple product motive M attached to E, which is a 8-dimensional motive over $\mathbb{Q}$. Assume that the functional equation of $L(M, s)$ has sign -1. Under certain technical assumptions, one can construct a cohomology class in the p-adic Bloch-Kato Selmer group of M using Hirzebruch–Zagier cycles. We prove that if this class is non-trivial, then the Bloch–Kato Selmer group of M has dimension 1. A key ingredient in the proof is a congruence formula for the cohomology class at certain unramified level raising primes, whose proof uses the fine geometry of the supersingular locus of a Hilbert modular threefold at an inert prime. This is a joint work with Yifeng Liu.

Eric Urban
Title: Euler systems via congruences between modular forms.

Abstract: I will talk about a recent strategy for constructing Euler systems via a refined study of certain congruences between modular forms of various level and weights. In particular, I will explain how it gives a way to construct Euler systems of higher ranks when their existence is predicted by the conjectures of B. Perrin-Riou and also how to connect them to p-adic L-functions. If time allows it, I will discuss a particular example for which the technical difficulties inherent to this method can be solved.

European Research Council
Established by the European Commission

European Commission

Horizon 2020
European Union funding for Research & Innovation

This workshop has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreements No 290766 and No 682152)